OPTIMAL POWER USE SURFACE FOR DESIGN OF WATER DISTRIBUTION SYSTEMS

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ABSTRACT

This paper intends to bring forward a hydraulic based Water Distribution System (WDS) design methodology named Optimal Power Use Surface (OPUS). Its objective is to reach least-cost designs, executing a reduced number of iterations, and focusing on the setting-up of efficient ways in which energy is dissipated and flow is distributed. For its validation, the proposed algorithm was tested on three well known benchmark networks, frequently reported in the literature: Two-Loop, Hanoi and Balerma. When compared to results obtained through other methodologies, OPUS stands out for allowing designs with constructive costs very similar to those obtained in previous works but requiring a number of iterations several orders of magnitude bellow, especially for real-size networks. The methodology proved that following hydraulic principles is an excellent choice to design WDS and also provides an alternative path to the tiresome search process undertaken by metaheuristics.

INTRODUCTION

The optimized design of WDSs is a relevant problem at a global scale, this being due to the scarcity of resources and the importance of drinking water for human life. This issue aggravates in the context of developing countries, where millions of people still suffer the lack of an adequate service. In this context minimum-cost design methodologies are essential.

Even though the design of WDSs is supposed to consider different criteria besides the construction costs (e.g. reliability, environmental impact and water quality), the minimum cost as the only objective is still used to validate and compare new design algorithms. This type of design consists in determining the diameter size of each pipe of the system in such a way that flow demands are satisfied with an adequate pressure and with a minimum capital cost. In spite of the fact that pipes are usually manufactured in discrete-sized diameters, the amount of possible pipe configurations is immense, which means that the problem is highly indeterminate. In fact, Yates et al. (1984) showed that it is a NP-HARD problem and thus only approximate methods could be successful in finding adequate solutions.

Initial approximations involved traditional optimization techniques such as enumeration, linear and non-linear programming. But more recently these have been replaced by different metaheuristic algorithms due to their ease of implementation and other advantages like their broader search of the solution space, a relatively small reliance on the system's initial configuration, and their capability of incorporating the discrete-sized diameters restriction. Successful attempts include Genetic Algorithms (Savic and Walters, 1997), Harmony Search (Geem, 2006), Scatter Search (Lin et al., 2007), Cross Entropy (Perelman and Ostfeld, 2007), Simulated Annealing (Reca et al., 2007), and Particle Swarm (Geem, 2009) among others.

These metaheuristics consist in bio-inspired algorithms that randomly generate a large number of possible solutions and test their fitness in terms of quality and capital costs. Generic learning functions are used to progressively improve the previous results. In the WDS design context, each solution corresponds to an alternative design, which means a different set of pipe diameter sizes.

The evaluation of each of the alternative designs requires running static hydraulic simulations, thus a large number of iterations is needed before convergence is reached. This makes metaheuristics very demanding in terms of computational effort regardless their flexibility and their capability of accomplishing near-optimal results. For this reason, apart from the cost of the final solution, the number of hydraulic simulations (or iterations) is the main indicator used to measure and compare the efficiency of the different methodologies. Even though the learning functions used in metaheuristic algorithms involve testing the hydraulic performance of each of the candidate solutions, neither of them make use of additional hydraulic criteria.

As a response to these tedious algorithms, some researchers have come through with new approaches that seek to develop a hydraulic treatment of the problem. While metaheuristics intend to optimize an objective function behaving towards the optimization variables simply as a series of numbers that must follow certain logic, without any understanding of the machinery behind that logic; these new approaches try to characterize the behaviour of the different hydraulic variables and understand the underlying dynamics.

In 1975 I-Pai Wu carried out an analysis for the drip irrigation main line design problem, considering the hydraulic principles that it follows. After setting up a minimum pressure ($P_{mín}$) at the end of the line, still a big number of configurations could be constructed. Wu discovered that each of these configurations lead to a different way in which energy is spent. After analysing numerous alternatives he concluded that the least-cost alternative was that with a parabolic hydraulic gradient line (HGL) with a sag of 15% of the total head-loss (H). Thus, optimal designs could be obtained by computing objective head-loss values for each pipe derived from the HGL fabricated using Wu´s criterion.

Later in 1983, Professor Ronald Featherstone from Newcastle University in the United Kingdom first proposed to extend Wu´s criterion to the optimization of looped networks. This idea seemed like a sound possibility and was further developed by Saldarriaga (1998), who analysed hydraulic gradient surfaces on several WDS designs obtained using metaheuristic algorithms. Based on Wu's criterion and Featherstone's idea, the works of Villalba (2004) and Ochoa (2009) proved that hydraulic criteria could be used as the basis of WDS design in order to replace the iteration-intensive stochastic approach required by metaheuristics; obtaining promising results, not only in performance, but also in the insight of the inner mechanics that govern WDS design.

Based on the works made by Ochoa (2009) and Villalba (2004), a first design methodology was developed by the CIACUA (Water Distribution and Sewer Systems Research Centre), named SOGH. It was tested on three well known benchmark networks (Two-Loop, Hanoi and Balerma). This paper intends to bring forward an improvement to this WDS design methodology named Optimal Power Use Surfaces (OPUS), which proposes a net hydraulic approach following the ideas of the aforementioned authors (Takahashi et al., 2010). The objective of this methodology is to reach least-cost designs with a reduced number of iterations especially for real-size networks. This can be accomplished through the use of deterministic hydraulic principles drawn from the analysis of flow distribution and the way energy is used in the systems. These principles provide an alternative path to the tiresome search process undertaken by metaheuristics. Each of the steps that make up the OPUS algorithm are explained in the following section. Three different benchmark problems (Two-Loop, Hanoi and Balerma) are employed to test the methodology and the results are presented and discussed in this paper. Finally, conclusions are drawn from the outcome obtained and future work guidelines are stated.

METHODOLOGY

The developed OPUS design methodology consists in 6 basic sub-processes which are shown in Figure 1 and explained below in this section.

Sump Search or Tree Structure. This step is based on two fundamental principles: The first one states that a WDS of minimum cost should convey the water to each of the demand nodes from the water sources, through a single route. This is drawn from the fact that redundancy is hydraulically inefficient, even though it favors reliability. Therefore, open WDSs could be a lot cheaper than looped networks, reason why this sub-process intends to decompose the looped system into an open tree-like structure (a spanning tree), in order to identify the nodes in the original model that correspond to the sumps of the open network (i.e., nodes with a lower head than that of all of its neighbors).

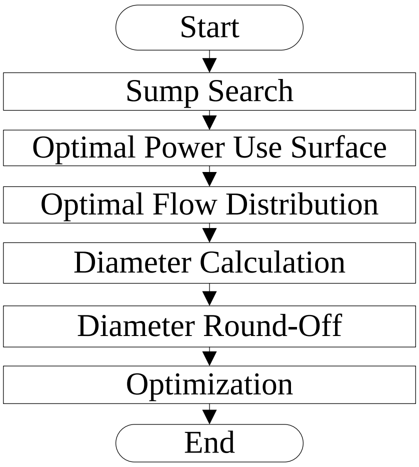

Figure 1: OPUS methodology BPMN diagram.

The second principle follows from the flow expression derived from the Darcy-Weisbach and Colebrook-White equations. Leaving all the other parameters constant, the flow (Q) presents a relation approximately proportional with the diameter to a power of 2.6. Assuming a standard pipe cost equation and replacing the diameter according to this proportion, the cost per length of a pipe as a function of its design flow behaves as shown in Figure 2; which means that as the design flow for a pipe increases, the marginal cost decreases.

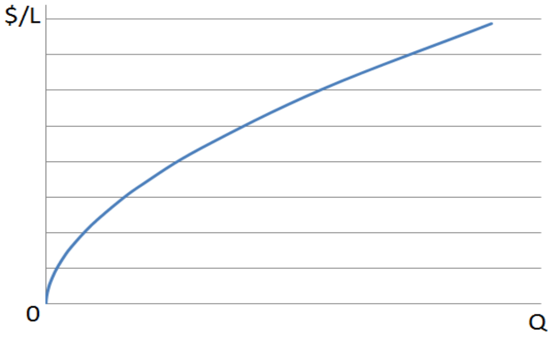

Figure 2: Schematic relation between pipe cost and flow.

From the abovementioned principles, an algorithm was designed in order to obtain the tree structure, aggregating flow values in the least number of main routes possible. The open network is set up starting from the water sources and then adding adjacent pipe-node pairs, one at a time. The group of available pairs in each iteration conform the 'search front' and each of these pairs are assigned a cost-benefit value $(B/\$)$, making up a recursive process.

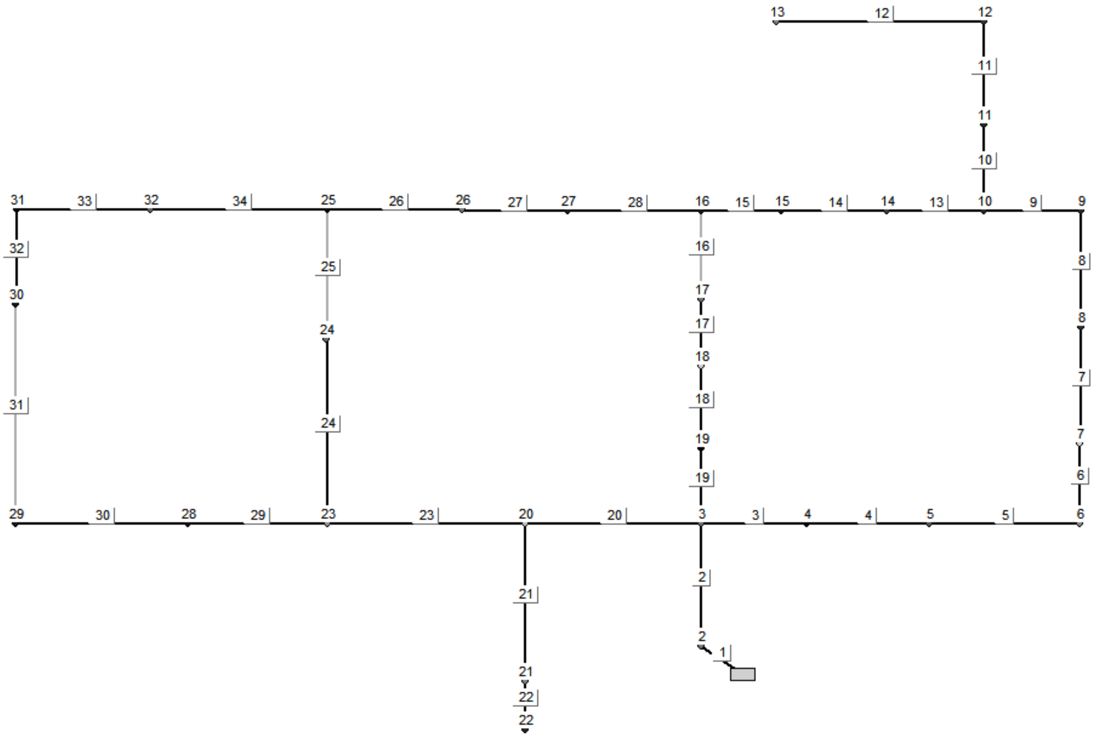

Figure 3: Layout of the Hanoi WDS. The labels show pipe and nodal identification numbers.

For example, take the Hanoi benchmark WDS shown in Figure 3: Starting from the source, the first pair to be added is the one consisting in pipe 1 and node 2 (<1, 2>). Then, the pair <2, 3> is added. At this point the pairs <3, 4>, <19, 19> and <20, 20> can be selected. These constitute the search front. Figure 3 shows the result for the entire execution of the sub-process, where the pipes highlighted (solid black) constitute the corresponding tree structure.

The pair in the front with the higher cost-benefit value is selected to be part of the tree structure. The cost-benefit function of a pair is calculated by computing the quotient between the demand of the new node and the marginal cost of connecting it to the source: This entails the addition of the total cost of the pair's pipe to the cost difference of transporting the additional flow through all of the upstream pipes. It is worth noting that these are not actual costs but proportional values drawn from the relation shown in Figure 2. The construction of the tree using this cost-benefit function has an $O(NN^2)$ time complexity, where NN is the number of nodes.

The cost-benefit function is used because it favours the creation of few main routes that transport the largest portion of the total water volume. The process concludes when all of the system nodes have been added to the tree structure and at the end the leaf nodes in the tree structure are assigned the status of 'sumps'.

Optimal Power Use Surface. This sub-process gives the name to the entire methodology, being essential to it due to the close relation that it has with the work developed by I-Pai Wu. In this step a set of objective hydraulic heads (also understood as objective head losses) is established for each pipe within the system. By analysing the behaviour of optimal designs, Ochoa (2009) corroborated Wu's suggestion, finding that the optimal hydraulic gradient line for pipe series was in effect a parabola. Besides, Ochoa discovered that the sag of this parabola depended on the demand distribution, the ratio between flow demands and pipe length, and the cost function; putting forward a methodology for its calculation.

Contrarily to the approaches made by Villalba (2004) and Ochoa (2009), in the proposed methodology, the optimal power use surface is computed in the tree structure instead of using a graph algorithm on the original network. In this sense, Ochoa's parabolic hydraulic gradient line is applied to each of the branches in the open network. First, the minimum allowable pressure is assigned to each of the sump nodes. Then, the topological distance for every node in the system to its source is calculated. Knowing the head in the reservoir, the heads of the intermediate nodes in each branch are calculated with a Parabolic Head Gradient Line (HGL) as a function of their distance to the source, as shown in Figure 4.

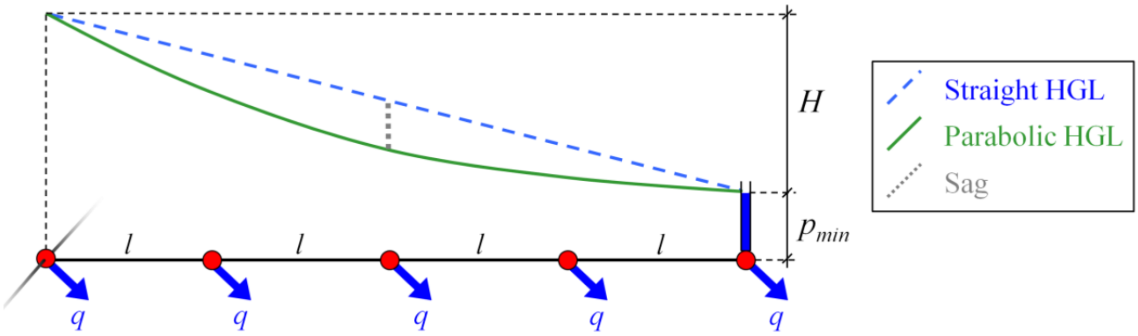

Figure 4: I-Pai Wu's criterion for predefining the head on each node.

As the branches converge while going over the tree upstream, it is necessary to recalculate the sag at each intersection by weighting the flow on each downstream route. It must be taken into account that the objective parabola has to be modified in the upstream direction, in case of encountering particularly elevated nodes, to make sure that the assigned head values meet the pressure requirements in every instance. Once this sub-process is concluded, all nodes must have an objective head value assigned, thus a flow is required in order to calculate the diameter of each pipe in the network.

Figure 5 shows an example of the optimal power use surface for the Hanoi network. Note that for the water source the HGL corresponds to the total head available in the reservoir and for the sump nodes it was assigned the minimum pressure.

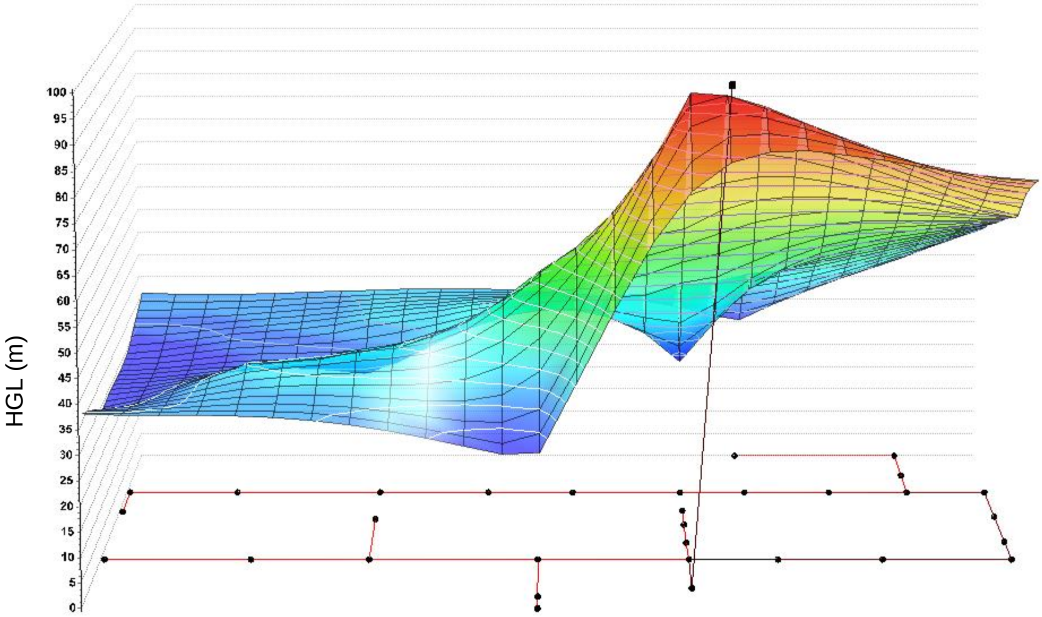

Figure 5: Assigned surface for the Hanoi network.

Optimal Flow Distribution. This step assigns a design flow to every pipe in the system. Considering that in a looped network a specific hydraulic gradient surface could be obtained by an infinite number of continuous diameter configurations (Saldarriaga et al., 2011), it is necessary to predefine an objective flow for each pipe in order to obtain a configuration that minimizes costs. Therefore, this sub-process pretends to find a unique flow distribution scheme that respects mass conservation and conforms to the optimal power use surface previously obtained. In this case, the process is executed using the original graph instead of the spanning tree.

Starting with the sumps, the flow demand is divided into the upstream pipes according to the following criterion: Only one pipe will be assigned the largest flow value, while the others will be assigned the flow that corresponds to the minimum diameter available ($d_{mín}$). In order to determine the principal pipe (i.e. the pipe that will have the largest portion of the total flow demanded), several criterions can be used to evaluate their fitness. Same as in the tree structure step, the function H/L^2 is used, which means that the pipe with the biggest value of this function will be defined as the principal pipe. For non-sump nodes, the total demand is calculated adding its own flow demand and the flow demanded downstream. An iterative-recursive algorithm (IRA) can be used to perform all of the calculations with an *O(NN)* time complexity. At the end of the process, all of the pipes in the system must have been assigned an objective flow value. Note that this step has a higher impact in the case of looped systems.

As an example, Figure 6 shows the optimal flow distribution that corresponds to the Hanoi network.

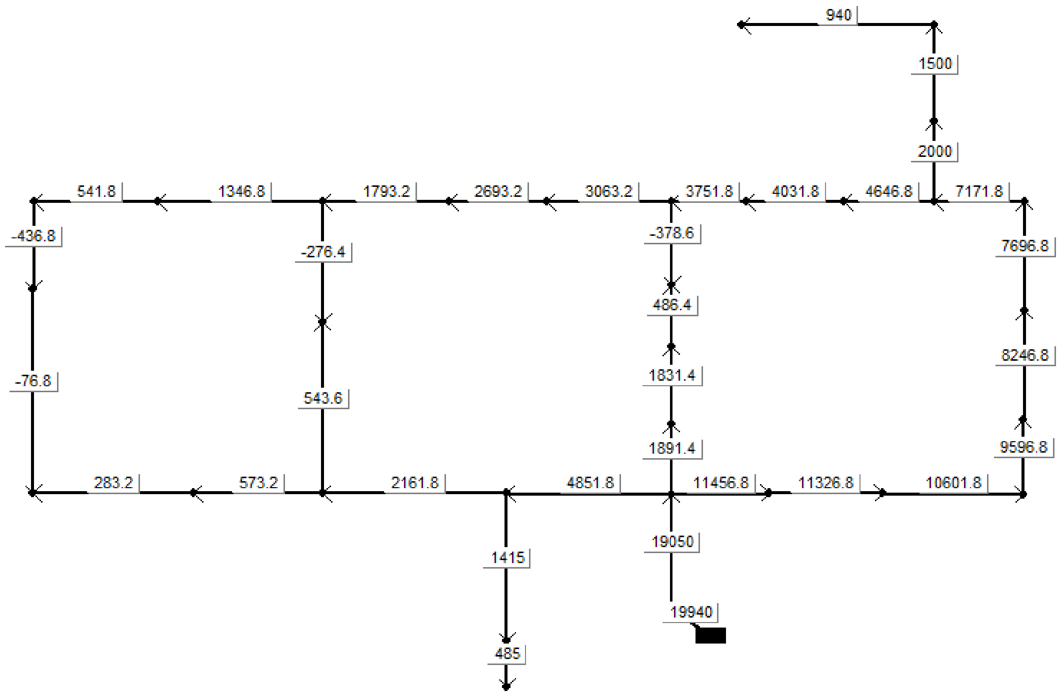

Figure 6: Optimal flow distribution for the Hanoi network (Flow rates in m^3/h).

Diameter Calculation. This sub-process assigns continuous diameter sizes to all pipes. Having predefined the objective head losses and the design flow rate for each pipe in the system, the continuous diameter needed is given by a straightforward calculation. This calculation is explicit when the Hazen-Williams equation is used and iteratively for the Darcy-Weisbach and the Colebrook-White equations. The resulting continuous design is in theory a full-operational WDS, with a cost very close to the minimum. Due to the limited availability of diameter sizes, a next step is required to transform this "optimal" design to a feasible one.

Diameter Round-Off. This step consists in approximating each continuous diameter to a discrete value from the list of commercially available diameter sizes, which is represented by the set $\mathbb{D} = \{D_1, \dots, D_{ND}\}$. It was found that rounding to the nearest equivalent flow value offers the best results, even though it can be done following several criterions. This is done by elevating the diameter values to a power of 2.6, as explained in the Tree Structure step. Unfortunately, this step affects drastically the system's hydraulic behaviour, especially if all the diameter sizes are rounded up or down.

Optimization

This final sub-process has two main goals: The first one is to ensure every node has a pressure higher than or equal to $\mathrm{P_{mín}}$; secondly, it seeks for possible cost reductions. Several criteria could be used to establish the order in which pipes diameter values must be increased. It was found that the pipes with larger unit head-loss difference between real and objective values should be changed first. The process must continue until the whole system has acceptable pressures. The second part executes a two-way sweep starting from the reservoirs going towards the sumps in the direction of the flow, and then backwards: The reduction of each pipe's diameter is considered twice. If any of these changes entails a pressure deficit it must be reversed immediately, otherwise it holds. To make sure minimum pressure is not being violated numerous hydraulic simulations are required.

In first place, the diameter size of one pipe is increased iteratively while there are nodes with pressure deficit. Thus, this sub-process requires the most number of iteration of the whole methodology, being necessary to run a hydraulic simulation per pipe, for each single diameter modification. This sole heuristic can be used alone to obtain sound designs, in spite of this, it is strongly dependant on the initial pipe configuration.

RESULTS

The OPUS methodology was used on three benchmark systems: Two-Loop, Hanoi and Balerma. Different configurations of the parameters defined on each step of the methodology were tested, looking for optimal designs with discrete diameters ($\mathbb{D} = \{D_1, \dots, D_{ND}\}$ contains only diameter available at the local market) and, in some cases, continuous ones ($\mathbb{D} = \mathbb{R}^+$) as potentially near optimal starting designs for the Round-off and Optimization sub-processes.

Two-Loop

Detailed information about this WDS can be found in Alperovits and Shamir (1977). The Hazen-Williams head-loss equation was used with a roughness coefficient $C = 130$, as specified in the mentioned publication, and also the unit prices table for each diameter size was adopted. With these values and with a minimum allowable pressure for every node of 30 m, the WDS was designed using the SOGH algorithm which is the OPUS predecessor methodology.

The optimal discrete design was reached after 51 hydraulic simulations which leaded to a $419,000 network. The number of hydraulic simulations reported by other authors who also reached this cost is shown in Table 1.

Table 1: Reported number of iterations before reaching a cost of $419.000 for the Two-Loop WDS.

Algorithm	**Number of iterations**
Genetic algorithms (Savic & Waters, 1997)	65,000
Simulated annealing (Cunha & Sousa, 1999)	25,000
Genetic algorithms (Wu & Simpson, 2001)	7,467
Shuffled frog leaping (Eusuff & Lansey, 2003)	11,155

Shuffled complex evolution (Liong & Atiquzzaman, 2004)	1,019
Genetic algorithms (Reca & Martínez, 2006)	10,000
Particle swarm optimization (Suribabu, 2006)	5,138
Harmony search (Geem, 2006)	1,121
Cross entropy (Perelman & Ostfeld, 2007)	35,000
Scatter search (Lin et al., 2007)	3,215
Particle swarm harmony search (Geem, 2009)	204
Differential evolution (Suribabu C. , 2010)	4,750
Honey-bee mating optimization (Mohan, 2010)	1,293
SOGH (Ochoa, 2009)	51

Hanoi

The Hanoi network was first presented by Fujiwara and Khang (1990) and similarly to Two-Loop network, it has become a well-known benchmark WDS. The head-loss equation commonly used is Hazen-Williams with a $C = 130$, the minimum pressure for the design scenario is 30 m and the pipes' costs can be calculated using a potential function of the diameter with a unit coefficient of \$1.1/m and an exponent of 1.5.

The least cost continuous design reached for Hanoi presents a cost of \$5,456,806, and is shown in Figure 7. It is worth noting that the continuous design involves diameter values higher than 40 inches, which corresponds to the maximum allowable diameter according to Fujiwara and Khang (1990). Therefore, two different discrete designs were performed: The first one being based on the original diameter list, and the second one considering the availability of a 50 inches diameter.

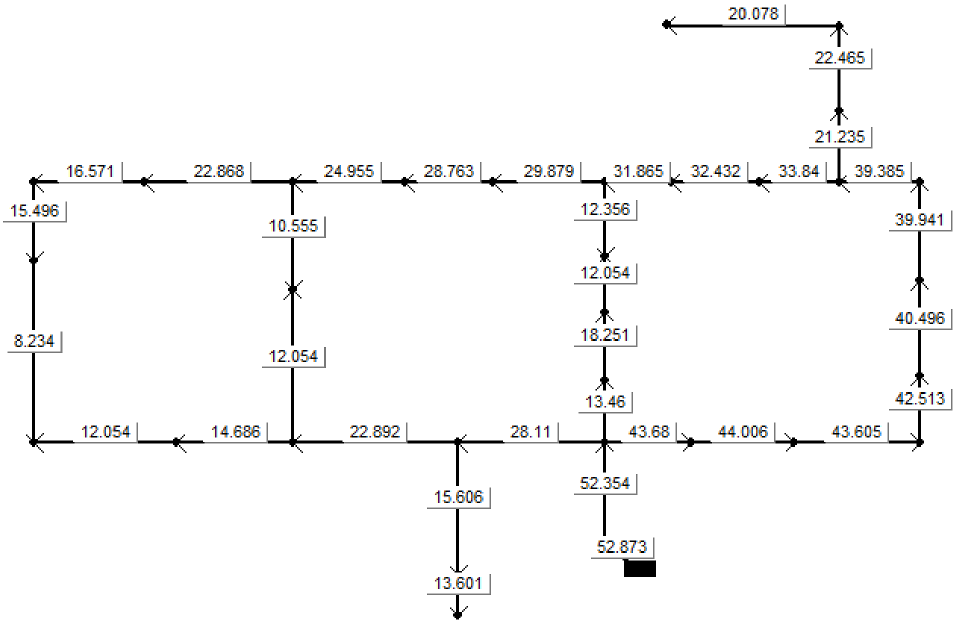

Figure 7: Hanoi network least cost continuous design (diameters in inches).

The OPUS methodology for the first case reached a cost of \$6'147,882.45 after 83 iterations. Although this is not the least cost reported, the number of hydraulic simulations needed to reach this result is three orders of magnitude smaller than that of other approaches, as can be seen in *Table 2*. The pipe diameter sizes in inches for this configuration are: 40, 40, 40, 40, 40, 40, 40, 40, 40, 30, 24, 24, 16, 16, 12, 12, 20, 20, 24, 40, 20, 12, 40, 30, 30, 20, 16, 12, 16, 12, 12, 12, 16 and 30 (these diameters are shown in order of pipe identification number).

Table 2: Reported costs and number of iterations for the Hanoi WDS.

Algorithm	**Cost (millions)**	**Number of iterations**
Genetic Algorithm (Savic and Walters, 1997)	$6.073	1,000,000
Simulated annealing (Cunha and Sousa, 1999)	$6.056	53,000
Harmony search (Geem, 2002)	$6.056	200,000
Shuffled frog leaping (Eusuff and Lansey, 2003)	$6.073	26,987
Shuffled complex evolution (Liong & Atiquzzaman, 2004)	$6.220	25,402
Genetic Algorithm (Vairavamoorthy, 2005)	$6.056	18,300
Ant colony optimization (Zecchin et al., 2006)	$6.134	35,433
Genetic Algorithms (Reca & Martínez, 2006)	$6.081	50,000
Genetic Algorithms (Reca et al., 2007)	$6.173	26,457
Simulated annealing (Reca et al., 2007)	$6.333	26,457
Simulated annealing with tabu search (Reca et al., 2007)	$6.353	26,457
Local search with simulated annealing (Reca et al., 2007)	$6.308	26,457
Harmony search (Geem, 2006)	$6.081	27,721
Cross entropy (Perelman & Ostfeld, 2007)	$6.081	97,000
Scatter search (Lin et al., 2007)	$6.081	43,149
Modified GA 1 (Kadu, 2008)	$6.056	18,000
Modified GA 2 (Kadu, 2008)	$6.190	18,000
Particle swarm harmony search (Geem, 2009)	$6.081	17,980
Heuristic based approach (Mohan S. a., 2009)	$6.701	70
Differential evolution (Suribabu C. , 2010)	$6.081	48,724
Honey-bee mating optimization (Mohan, 2010)	$6.117	15,955
Heuristic based approach (Suribabu C. , 2012)	$6.232	259
SOGH (Ochoa, 2009)	$6.337	94
Optimal power use surface (this study)	$6.173	83

Extrapolating the cost function for a 50” diameter it would have a unit cost of $388.91/m. Taking this into account, the total cost of the design obtained following the OPUS algorithm was of only $5’342,840.13, as the real hydraulic gradient surface resembled the optimal in a higher degree. The diameter sizes in inches are: 50, 50, 40, 40, 40, 40, 40, 24, 24, 40, 20, 20, 12, 12, 12, 16, 20, 20, 20, 40, 16, 12, 40, 30, 24, 20, 12, 12, 12, 12, 12, 12, 12 and 20.

Balerma

Balerma corresponds to a WDS of an irrigation district in Almería, Spain. The pipe diameter sizes commercially available for its design are manufactured exclusively in PVC, with an absolute roughness coefficient of 0.0025 mm. The minimum pressure allowable is of 20 m and the pipes’ costs are calculated using a potential function, with a power of 2.06. Its topology is presented in Figure 8.

Figure 8: Topology of the Balerma network.

As a result of implementing the OPUS methodology on this network, a €1.755 millions continuous design was founded. Also, after executing the round-off and optimization processes, the optimal discrete design was reached requiring 957 hydraulic simulations which leaded to a €2.106 millions network. Table 3 presents other reported costs and their respective number of iterations.

Table 3: Reported costs and number of iterations for the Balerma WDS.

Algorithm	Cost (€ millions)	Number of iterations
Genetic algorithm (Reca & Martínez, 2006)	2.302	10.000.000
Harmony search (Geem, 2006)	2.601	45.400
Harmony search (Geem, 2006)	2.018	10.000.000
Genetic algorithm (Reca et al., 2007)	3.738	45.400
Simulated annealing (Reca et al., 2007)	3.476	45.400
Simulated annealing with taboo search (Reca et al., 2007)	3.298	45.400
Local search with simulated annealing (Reca et al., 2007)	4.310	45.400
Hybrid discrete dynamically dimensioned search (Tolson, 2009)	1,940	30,000,000
Harmony search with particle swarm (Geem, 2009)	2.633	45.400

SOGH (Ochoa, 2009)	2.100	1.779
Memetic algorithm (Baños, 2010)	3,120	45,400
Genetic heritage evolution by stochastic transmission (Bolognesi, 2010)	2,002	250,000
Differential evolution (Zheng, 2012)	1,998	2,400,000
Self-adaptive differential evolution (Zheng, 2012)	1,983	1,300,000
Optimal power use surface (this study)	2.106	957

CONCLUSIONS

The WDS least-cost design methodology known as Optimal Power Use Surface (OPUS) herein introduced, considers optimal distribution pattern of flow in the network as a way of spending energy properly. This approach differentiates it from metaheuristic algorithms that explore the solution space without considering hydraulic principles.

The methodology significantly reduces the number of iterations and keeps the constructive costs of the network significantly close to the minimum. In the case of Hanoi the difference results only of 1.9% with respect to the lowest cost reported in the literature and with a number of iterations three orders of magnitude below.

OPUS is a methodology that worked well on large networks, making it possible to minimize constructive costs in a very reduced number of iterations. On the other hand, when applied to small WDSs, the hydraulics result very affected due to the relative difference between the tree structure and the real looped network. Taking into account that the design is executed based on the open network (spanning tree), the final result ends up being governed by the optimization process and not by the steps based on hydraulic principles.

This methodology clearly proves that considering hydraulic bases allows the optimization of WDS design to reduce significantly the number of iterations required. Theoretical networks with some restrictions that limit the design possibilities were tested in this paper. For this reason, it is recommended to test this methodology on real networks and including different additional objectives such as maximizing reliability and minimizing leakage. Finally energy approaches can also be applied to calibrate models and design system operation.

REFERENCES

Alperovits, E., and Shamir, U. (1977) Design of optimal water distribution systems. Water Resources Research, Vol.13, No.6, pp. 885-900.

Baños, C. G. (2010). A memetic algorithm applied to the design of water distribution networks. *Applied Soft Computing*, 261-266.

Bolognesi, A. e. (2010). Genetic Heritage Evolution by Stochastic Transmission in the optimal design of water distribution networks. *Advances in Engineering Software*, 792-801.

Cunha, M. a. (1999). Water distribution network design optimization: Simulated annealing approach. *J. Water Resour. Plan. Manage.* , 215-221.

Eusuff, M. a. (2003). Optimization of water distribution network design using the shuffled frog leaping algorithm. *J. Water Resour. Plan. Manage.* , 210-225.

Fujiwara, O. and Khang, D. (1990). A two phase decomposition methods for optimal design of looped water distribution networks. *Water Resources Research*, Vol.26, No. 4, pp. 539-549.

Geem, Z. K. (2002). Harmony search optimization: Application to pipe network design. *Int. J. Model. Simulat.* , 125-133.

Geem, Z. K. (2006). Optimal cost design of water distribution networks using harmony search. *Engineering Optimization*, Vol.38, No.3, pp. 259-277.

Geem, Z. K. (2009). Particle-swarm harmony search for water network design. *Engineering Optimization*, Vol.41, No.4, pp. 297-311.

Kadu, M. R. (2008). Optimal design of water networks using a modified genetic algorithm with reduction in search space. *J. Water Resour. Plan. Manage.* , 147-160.

Lin, M., Liu, G. and Chu, C., (2007) Scatter search heuristic for least-cost design of water distribution networks, *Engineering Optimization*, Vol.39, No.7, pp. 857-876.

Liong, S. and Atiquzzaman, M. (2004) Optimal design of water distribution network using shuffled complex evolution. *Journal of the Institution of Engineers*, Vol.44, No.1, pp. 93-107.

Mohan, S. a. (2010). Optimal water distribution network design with Honey-Bee mating optimization. *J. Water Resour. Plan. Manage*., 117-126.

Mohan, S. a. (2009). Water distribution network design using heuristics-based algorithm. *J. Comput.Civ. Eng.*, 249-257.

Ochoa, S. (2009). Optimal design of water distribution systems based on the optimal hydraulic gradient surface concept. MSc Thesis, dept. of Civil and Environmental Engineering, Universidad de los Andes, Bogotá, Col. (In Spanish).

Perelman, L. and Ostfeld, A. (2007). An adaptive heuristic cross entropy algorithm for optimal design of water distribution systems. *Engineering Optimization*, Vol.39, No.4, pp. 413-428.

Reca, J. and Martínez, J. (2006). Genetic algorithms for the design of looped irrigation water distribution networks. *Water Resources Research*, Vol.44, W05416.

Reca, J., Martínez, J., Gil, C. and Baños, R. (2007). Application of several meta-heuristic techniques to the optimization of real looped water distribution networks. *Water Resources Management*, Vol.22, No.10, pp. 1367-1379.

Saldarriaga, J. (1998 and 2007). *Hidráulica de Tuberías. Abastecimiento de Agua, Redes, Riegos*. Ed. Alfaomega. Ed. Uniandes. ISBN: 978-958-682-680-8.

Saldarriaga, J., Takahashi, S., Hernández, F. and Escovar, M. (2011). Predetermining pressure surfaces in water distribution system design. In *Proceedings of the World Environmental and Water Resources Congress 2011, ASCE*.

Savic, D. and Walters, G. (1997). Genetic algorithms for least cost design of water distribution networks. *J. Water Resour. Plan. Manage*., 67-77.

Suribabu, C. a. (2006). Design of water distribution networks using particle swarm optimization. *Urban Water Journal*, 111-120.

Suribabu, C. (2010). Differential evolution algorithm for optimal design of water distribution networks. *J. of Hydroinf*., 66-82.

Suribabu, C. (2012). Heuristic Based Pipe Dimensioning Model for Water Distribution Networks. *Journal of Pipeline Systems Engineering and Practice*, 45 p.

Takahashi, S., Saldarriaga, S., Hernández, F., Díaz, D. and Ochoa, S. (2010). An energy methodology for the design of water distribution systems. In *Proceedings of the World Environmental and Water Resources Congress 2010, ASCE*.

Tolson, B. A. (2009). Hybrid discrete dynamically dimensioned search (HD-DDS) algorithm for water distribution system design optimization. *Water Resources Research*, 45 p.

Vairavamoorthy, K. a. (2005). Pipe index vector: A method to improve genetic-algorithm-based pipe optimization. *J. Hydraul. Eng.*, 1117-1125.

Villalba, G. (2004). Optimal combinatory algorithms applied to the design of water distribution systems. MSc Thesis, dept. of Systems and Computation Engineering, Universidad de los Andes.

Wu, I. (1975). Design of drip irrigation main lines. *Journal of Irrigation and Drainage Division*, Vol.101, No.4, pp. 265-278.

Wu, Z. and Simpson, A. (2001). Competent genetic evolutionary optimization of water distribution systems. *Journal of computing in civil engineering*. Vol.15, No.2, 2001, pp. 89-101.

Yates, D., Templeman, A. and Boffey, T. (1984). The computational complexity of the problem of determining least capital cost designs for water supply networks. *Engineering Optimization*, Vol. 7, No.2, pp. 142-155.

Zecchin, A., Simpson, A., Maier, H., Leonard, M., Roberts, A., and Berrisfors, M. (2006) Application of two ant colony optimization algorithms to water distribution system optimization. *Mathematical and Computer Modeling*, Vol.44, No. 5-6, pp. 451-468.

Zheng, F. e. (2012). A Self-Adaptive Differential Evolution Algorithm Applied to Water Distribution System Optimization. *Journal of Computing in Civil Engineering*, 45 p.