# OPTIMAL DESIGN OF WATER DISTRIBUTION NETWORKS USING MOCK OPEN TREE TOPOLOGY

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## ABSTRACT
This paper describes a new approach given to the Optimal Power Use Surface (OPUS) methodology, which consists of the decomposition of a Water Distribution System (WDS) into an open tree-like structure (a spanning tree). Once the sumps in the model are identified, Integer Linear Programming (ILP) is used to accelerate the design process, calculating the diameter of every node in the tree. This is achieved by focusing on the setting-up of efficient ways in which energy is dissipated and flow is distributed. The tree structure is built starting from the water sources. Then, the rest of the tree is assembled adding adjacent pipe-node pairs, one at a time. The methodology is tested on three benchmark problems (Hanoi, Balerma and Taichung). When compared to results obtained through other methodologies, this new approach stands out for allowing designs with constructive costs very similar to those obtained in previews works but requiring a number of iterations several orders of magnitude bellow. The methodology proves that following hydraulic principles and applying ILP is an excellent choice to obtain low-cost WDS designs, with very little effort and providing an alternative path to the tiresome search process undertaken by metaheuristics.

## INTRODUCTION
Obtaining optimal designs of WDSs is a problem of great importance at a global scale. This is due to the scarcity of resources available to solve this issue and the fact that water supply is essential for human life. The problem becomes critical in the context of developing countries, where millions of people still suffer the lack of an adequate service. In places with this background, minimum-cost design methodologies become essential.

Even though the design of WDSs is supposed to consider different criteria besides the construction costs (e.g. reliability, environmental impact and water quality), the minimum cost as the only objective is still used to validate and compare new design algorithms. This type of design consists in determining the set of pipe diameter sizes that offers a minimum capital cost, satisfying flow demands with an adequate pressure. In spite of the fact that pipes are usually manufactured in discrete-sized diameters, the amount of possible pipe configurations is immense, which means that the problem is highly indeterminate. In fact, Yates et al. (1984) showed that it is a NP-HARD problem and thus only approximate methods could be successful in finding adequate solutions.

Initial approximations involved traditional optimization techniques such as enumeration, linear and non-linear programming. But more recently different metaheuristic algorithms have gained popularity due to their ease of implementation and other advantages like their broader search of the solution space, a relatively small reliance on the system's initial configuration, and their capability of incorporating the discrete-sized diameters restriction. Successful attempts include Genetic Algorithms (Savic and Walters, 1997), Harmony Search (Geem, 2006), Scatter Search (Lin et al., 2007), Cross Entropy (Perelman and Ostfeld, 2007), Simulated Annealing (Reca et al., 2007), and Particle Swarm (Geem, 2009) among others.

These metaheuristics consist in bio-inspired algorithms that randomly generate a large number of possible solutions and test their fitness in terms of quality and capital costs. Generic learning functions are used to progressively improve the previous results. In the WDS design context, each solution corresponds to an alternative design, which means a different set of pipe diameter sizes. The evaluation of each of the alternative designs requires running static hydraulic simulations, thus a large number of iterations is needed before convergence is reached. This makes metaheuristics very demanding in terms of computational effort regardless their flexibility and their capability of accomplishing near-optimal results. For this reason, apart from the cost of the final solution, the number of hydraulic simulations (or iterations) is the main indicator used to measure and compare the efficiency of the different methodologies. Even though the learning functions used in metaheuristic algorithms involve testing the hydraulic performance of each of the candidate solutions, neither of them make use of additional hydraulic criteria.

As a response to these tedious algorithms, some researchers have come through with new approaches that seek to develop a hydraulic treatment of the problem, taking into account that now that near-optimal WDS designs are readily available, the patterns behind these results and the hydraulic principles that they follow can be easily rebuilt through retrospection. While metaheuristics intend to optimize an objective function behaving towards the optimization variables simply as a series of numbers that must follow certain logic, without any understanding of the machinery behind that logic; these new approaches try to characterize the behaviour of the different hydraulic variables and understand the underlying dynamics.

In 1975 I-Pai Wu carried out an analysis for the drip irrigation main line design problem, considering the hydraulic principles that it follows. After setting up a minimum pressure ($P_{min}$) at the end of the line, still a big number of configurations could be constructed. Wu discovered that each of these configurations involved a different way of spending the energy available in the system. After analysing numerous alternatives he concluded that the least-cost alternative was that with a parabolic hydraulic gradient line (HGL) with a sag of 15% of the total head-loss ($H$). Thus, optimal designs could be obtained by computing objective head-loss values for each pipe derived from the HGL fabricated using Wu´s criterion.

Later in 1983, Professor Ronald Featherstone from Newcastle University in the United Kingdom first proposed to extend Wu´s criterion to the optimization of looped networks. This idea seemed like a sound possibility and was further developed by Saldarriaga (1998), who analysed hydraulic gradient surfaces on several WDS designs obtained using metaheuristic algorithms. Based on Wu's criterion and Featherstone's idea, the works of Villalba (2004) and Ochoa (2009) proved that hydraulic criteria could be used as the basis of WDS design in order to replace the iteration-intensive stochastic approach required by metaheuristics; obtaining promising results, not only in performance, but also in the insight of the inner mechanics that govern WDS design.

Based on the works developed by Ochoa (2009) and Villalba (2004), a first design methodology was developed by the CIACUA (Water Distribution and Sewer Systems Research Centre), named SOGH. It was tested on three well known benchmark networks (Two-Loop, Hanoi and Balerma). This methodology was then succeeded by the Optimal Power Use Surfaces (OPUS) methodology, which proposed a net hydraulic approach following the ideas of the aforementioned authors (Takahashi et al., 2010). The objective of this methodology is to reach least-cost designs with a reduced number of iterations especially for real-size networks. This can be accomplished through the use of deterministic hydraulic principles drawn from the analysis of flow distribution and the way in which energy is used along the systems. The latest approach of the OPUS methodology is the one presented in this paper, which incorporates the use of ILP in the former algorithm, with the purpose of accelerating the process. This can be done since one of OPUS' steps consists in transforming the looped network in an open structure, and the problem of the design of an open system has been previously solved using IPL principles (Alperovits & Shamir, 1977). The design of the open network is obtained straightforward and requires a total of ND (number of diameters commercially available) iterations, which are typically between 8 and 10. Unlike, the traditional OPUS methodology allows a first design without running any hydraulic simulation. In spite of this, the new approach is expected to allow a better initial design for the posterior optimization step, which contributes with the greatest number of iterations. Each of the sub-processes that make up the new alternative OPUS methodology are explained in the following section. The methodology is tested on three benchmark problems (Hanoi, Balerma and Taichung). Finally, conclusions are drawn from these results and their implications, and guidelines for future work are suggested.

## METHODOLOGY

The developed design methodology using mock open tree topology consists in 5 basic sub-processes which are shown in Figure 1 and explained below in this section. Note that the first and last sub-processes in Figure 1 are exactly the same than in the former OPUS methodology. However, there is an important variation in the middle steps of the algorithm since the new approximation includes the use of IPL in order to design the tree structure network obtained from the Sump Search step; instead of applying the optimal power use surface criterion and a subsequent optimal flow distribution.

**Sump Search or Tree Structure.** This step is based on two fundamental principles: The first one states that a WDS of minimum cost should convey the water to each of the demand nodes from the water sources, through a single route. This is drawn from the fact that redundancy is hydraulically inefficient, even though it favors reliability. Therefore, open WDSs could be a lot cheaper than looped networks, reason why this sub-process intends to decompose the looped system into an open tree-like structure (a spanning tree), in order to identify the nodes in the original model that correspond to the sumps of the open network (i.e., nodes with a lower head than that of all of its neighbors).

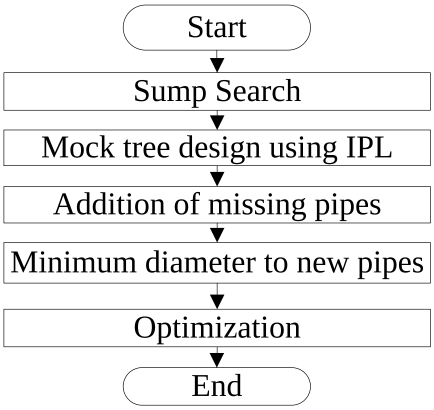


*Figure 1: Mock open tree methodology BPMN diagram.*

The second principle follows from the flow expression derived from the Darcy-Weisbach and Colebrook-White equations. Leaving all the other parameters constant, the flow ($Q$) presents a relation approximately proportional with the diameter to a power of 2.6. Assuming a standard pipe cost equation and replacing the diameter according to this proportion, the cost per length of a pipe as a function of its design flow behaves as shown in Figure 2; which means that as the design flow for a pipe increases, the marginal cost decreases.

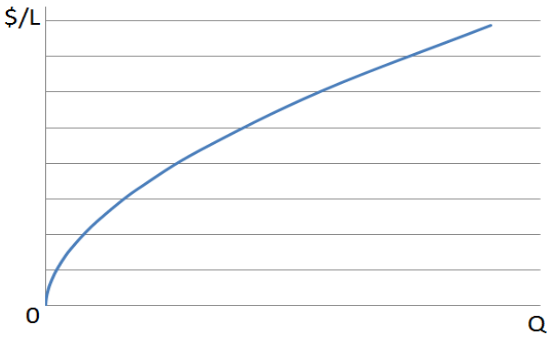


*Figure 2: Schematic relation between pipe cost and flow.*

From the abovementioned principles, an algorithm was designed in order to obtain the tree structure, aggregating flow values in the least number of main routes possible. The open network is set up starting from the water sources and then adding adjacent pipe-node pairs, one at a time. The group of available pairs in each iteration conform the 'search front' and each of these pairs are assigned a cost-benefit value ($B/\$$), making up a recursive process.

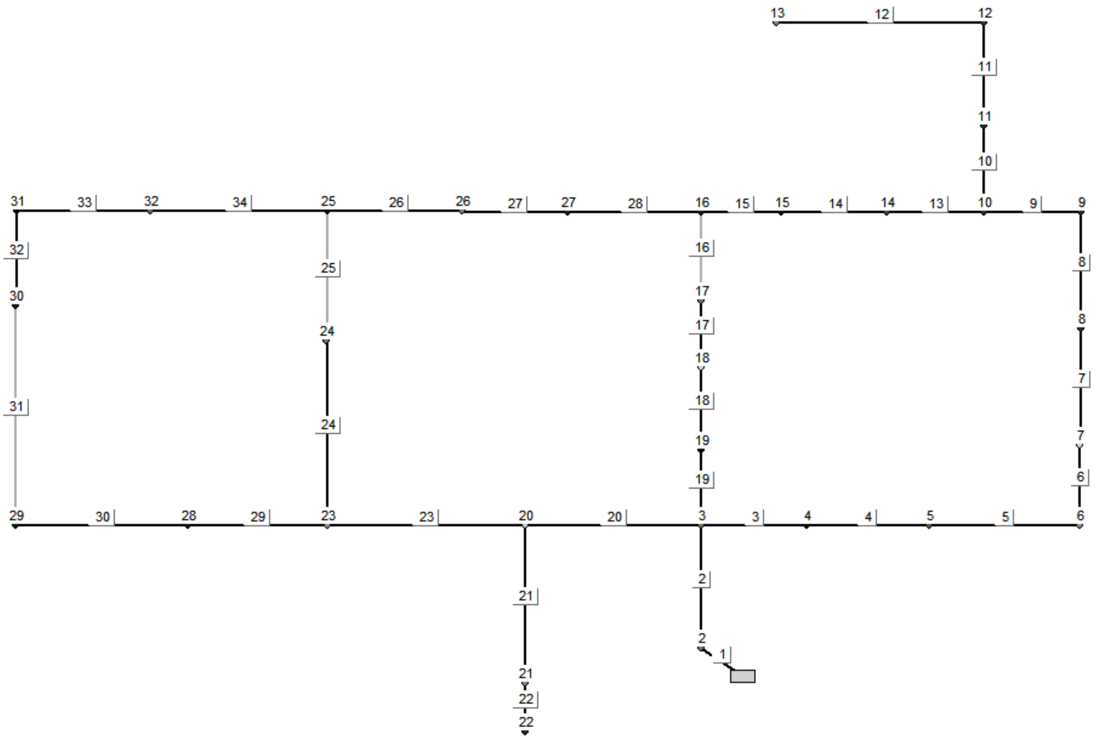

*Figure 3: Layout of the Hanoi WDS. The labels show pipe and nodal identification numbers.*

For example, take the Hanoi benchmark WDS shown in Figure 3: Starting from the source, the first pair to be added is the one consisting in pipe 1 and node 2 (<1, 2>). Then, the pair <2, 3> is added. At this point the pairs <3, 4>, <19, 19> and <20, 20> can be selected. These constitute the search front. Figure 3 shows the result for the entire execution of the sub-process, where the pipes highlighted (solid black) constitute the corresponding tree structure.

The pair in the front with the higher cost-benefit value is selected to be part of the tree structure. The cost-benefit function of a pair is calculated by computing the quotient between the demand of the new node and the marginal cost of connecting it to the source: This entails the addition of the total cost of the pair's pipe to the cost difference of transporting the additional flow through all of the upstream pipes. It is worth noting that these are not actual costs but proportional values drawn from the relation shown in Figure 2. The construction of the tree using this cost-benefit function has an $O(NN^2)$ time complexity, where NN is the number of nodes.

The cost-benefit function is used because it favours the creation of few main routes that transport the largest portion of the total water volume. The process concludes when all of the system nodes have been added to the tree structure and at the end the leaf nodes in the tree structure are assigned the status of ‘sumps’.

**Mock tree design using IPL.** This sub-process focuses in designing the tree structure that resulted from the previous step. The design is straightforward and is obtained applying the formulation presented in (Hernández, 2012) which is implemented in the software Xpress IVE:

- **Sets**

N: Set that contains all the nodes.
$D$: Set that contains all the commercially available diameters.

- **Decision variables**

$\mathrm{X_{ijd}}$: Binary variable.

$$\mathrm{X_{ijd}} = \begin{cases} 1 & \text{if the section between i } \in \text{N and j } \in \text{N has a diameter of d } \in \text{D} \\ 0 & \text{otherwise} \end{cases}$$

As it can be seen, de decision variable $\mathrm{X_{ijd}}$ can only take a value of 1 or 0; it will have a value of 1 if the model assigns a diameter of $d$ to the section between node i and node j. Additionally, it is necessary to define an auxiliary variable which contains the pressure of each node of the system.
p: Auxiliary decision variable that defines the pressure in node i.

- **Constraints of the problem**

Constraint of HGLmin: Constraint that guarantees a HGL equal or greater than a minimum in every node.

$$\mathrm{HGL}_i > HGL_{min_i} \qquad \forall\, \mathrm{i} \in \mathrm{N}$$

Constraint of HGL in downstream nodes: Constraint that guarantees that the HGL in node j $\in$ N downstream node i $\in$ N is equal to the HGL upstream minus the losses (dp) generated in the pipeline section between node i $\in$ N and node j $\in$ N provided that nodes $i$ and $j$ are linked.

$$\mathrm{HGL_j} = \mathrm{HGL_i} - \sum_{\mathrm{d}\in\mathrm{D}} \mathrm{dp_{ijd}} \cdot \mathrm{X_{ijd}} \qquad \forall\, \mathrm{i} \in \mathrm{N}, \forall\, \mathrm{j} \in \mathrm{N} \mid \exists\, \mathrm{w(i,j)}$$

where $HGL_j$corresponds to the hydraulic gradient line in node $j$ $\in$ N, which is downstream node i $\in$ N. On the other hand $\mathrm{dp_{ijd}}$ corresponds to the total head losses generated in the section between nodes i and j if a pipe of diameter d is used. $\mathrm{X_{ijd}}$ corresponds to the decision variable. It is worth noting that the values of $\mathrm{dp_{ijd}}$ correspond to the total head losses obtained as parameters of the problem, in the total head losses matrix.
Constraint of unique diameter in each section: This constraint guarantees that only one diameter is assigned to each section of the system.

$$\sum_{d \in D} X_{ijd} = 1 \qquad \forall\, i \in N, \forall\, j \in N \mid \exists\, w(i,j)$$

- **Objective function**

$$\sum_{i \in N} \sum_{j \in N} \sum_{d \in D} C_{ijd} \cdot X_{ijd}$$

where $C_{ijd}$ is the cost of using a diameter $d \in D$ in the section between node $i \in N$ and node $j \in N$. $X_{ijd}$ corresponds to the decision variable. The objective is to minimize this function. The constraints presented previously will be in charge of meeting the hydraulic requirements of minimum pressure. It is worth noting that the values of $C_{ijd}$ correspond to parameters of the objective function, which are obtained from the cost matrix.

Knowing the flow demand in every node of the network, the minimum pressure required ($P_{min}$) and the cost function; it is possible to calculate the cost matrix, total head losses matrix, connectivity matrix and the minimum HGLs. From this and knowing the head in the reservoir, Xpress gives as a result the minimum-cost design meeting the problem's restrictions. This step contributes with as many iterations as diameters are commercially available, due to the fact that in order to obtain the head losses matrix it is necessary to assign the same diameter to all the pipes in the system and execute a hydraulic simulation, this for every available diameter.

**Addition of missing pipes.** This step consists in adding to the tree structure the pipes that were removed from the original network in the first step, in order to obtain again the latter. This is the sub-process that allows the extension of the methodology using IPL to the design of looped networks. Even though the network designed through IPL is an open structure, this is later converted back again to a looped network to maintain the original topology.

**Minimum diameter to new pipes.** Due to the way in which the tree structure is generated, the open network represents adequately the original network's hydraulic behaviour, as long as the diameters are the same for the common pipes. Namely, the pressure in the nodes will be the same in both systems, since in theory the removed pipes don't convey water because they link two sumps in every case. This means that if the restriction of minimum pressure is fulfilled in the design of the tree structure, it will be met as well in the looped network, despite the diameter assigned to the new pipes. Given that the design obtained after the application of IPL is the optimum for the open network, it is possible to just assign the minimum diameter to the rest of the pipes, so that the capital cost of the network increases as least as possible; while the fulfilment of the discrete diameter restriction is guaranteed.

This is valid for a single network, which in this case corresponds to a system with only one reservoir. Reason why for networks with more than one water source it is necessary to make sure that each node has a pressure at least equal to $P_{min}$. This is accomplished in the optimization step, which is the final step of OPUS methodology, as well as the final step of the new approach presented in this paper.

**Optimization.** This final sub-process has two main goals: The first one is to ensure every node has a pressure higher than or equal to $P_{min}$; secondly, it seeks for possible cost reductions. Several criteria could be used to establish the order in which pipes diameter values must be increased. It was found that the pipes with larger unit head-loss difference between real and objective values should be changed first. The process must continue until the whole system has acceptable pressures. The second part executes a two-way sweep starting from the reservoirs going towards the sumps in the direction of the flow, and then backwards: The reduction of each pipe's diameter is considered twice. If any of these changes entails a pressure deficit it must be reversed immediately, otherwise it holds. To make sure minimum pressure is not being violated numerous hydraulic simulations are required.

In first place, the diameter size of one pipe is increased iteratively while there are nodes with pressure deficit. Thus, this sub-process requires the most number of iteration of the whole methodology, being necessary to run a hydraulic simulation per pipe, for each single diameter modification. This sole heuristic can be used alone to obtain sound designs, in spite of this, it is strongly dependant on the initial pipe configuration.

## RESULTS

The methodology methodology using mock open tree topology was used on three benchmark systems: Hanoi, Balerma and Taichung.

### Hanoi

The Hanoi network was first presented by Fujiwara and Khang (1990) and similarly to Two-Loop network, it has become a well-known benchmark WDS. The head-loss equation commonly used is Hazen-Williams with a $C = 130$, the minimum pressure for the design scenario is 30 m and the pipes' costs can be calculated using a potential function of the diameter with a unit coefficient of $1.1/m and an exponent of 1.5.

The Mock Tree methodology reached a cost of $6'163,754 after 119 iterations. Although this is not the least cost reported, the number of hydraulic simulations needed to reach this result is three orders of magnitude smaller than that of other approaches, as can be seen in *Table 1*. The pipe diameter sizes in inches for this configuration are: 40, 40, 40, 40, 40, 40, 40, 40, 40, 30, 30, 24, 20, 16, 12, 12, 16, 20, 20, 40, 20, 12, 40, 30, 30, 20, 12, 12, 16, 16, 12, 12, 16 and 24 (these diameters are shown in order of pipe identification number).

*Table 1: Reported costs and number of iterations for the Hanoi WDS.*

| **Algorithm** | **Cost (millions)** | **Number of iterations** |
|---|---|---|
| Genetic Algorithm (Savic and Walters, 1997) | $6.073 | 1,000,000 |
| Simulated annealing (Cunha and Sousa, 1999) | $6.056 | 53,000 |
| Harmony search *(Geem, 2002)* | $6.056 | 200,000 |
| Shuffled frog leaping (Eusuff and Lansey, 2003) | $6.073 | 26,987 |
| Shuffled complex evolution (Liong & Atiquzzaman, 2004) | $6.220 | 25,402 |
| Genetic Algorithm *(Vairavamoorthy, 2005)* | $6.056 | 18,300 |
| Ant colony optimization (Zecchin et al., 2006) | $6.134 | 35,433 |
| Genetic Algorithms (Reca & Martínez, 2006) | $6.081 | 50,000 |
| Genetic Algorithms (Reca et al., 2007) | $6.173 | 26,457 |
| Simulated annealing (Reca et al., 2007) | $6.333 | 26,457 |
| Simulated annealing with tabu search (Reca et al., 2007) | $6.353 | 26,457 |
| Local search with simulated annealing (Reca et al., 2007) | $6.308 | 26,457 |
| Harmony search (Geem, 2006) | $6.081 | 27,721 |

| | | |
|---|---|---|
| Cross entropy (Perelman & Ostfeld, 2007) | $6.081 | 97,000 |
| Scatter search (Lin et al., 2007) | $6.081 | 43,149 |
| Modified GA 1 *(Kadu, 2008)* | $6.056 | 18,000 |
| Modified GA 2 *(Kadu, 2008)* | $6.190 | 18,000 |
| Particle swarm harmony search (Geem, 2009) | $6.081 | 17,980 |
| Heuristic based approach *(Mohan S. a., 2009)* | $6.701 | 70 |
| Differential evolution *(Suribabu C. , 2010)* | $6.081 | 48,724 |
| Honey-bee mating optimization *(Mohan S. a., 2010)* | $6.117 | 15,955 |
| Heuristic based approach *(Suribabu C. , 2012)* | $6.232 | 259 |
| SOGH (*Ochoa*, 2009) | $6.337 | 94 |
| OPUS *(Saldarriaga, Páez, Cuero, & León, 2012)* | $6.173 | 83 |
| Mock Tree (this study) | $6.163 | 119 |

Extrapolating the cost function for a 50” diameter it would have a unit cost of $388.91/m. Taking this into account, the total cost of the design obtained following the Mock Tree algorithm was of only $5’414,077, with a total of 58 iterations. The diameter sizes in inches are: 40, 50, 40, 40, 40, 40, 30, 30, 30, 24, 24, 20, 16, 12, 12, 12, 16, 16, 20, 40, 16, 12, 30, 30, 30, 20, 12, 12, 16, 12, 12, 12, 16, and 20.

**Balerma**

Balerma corresponds to a WDS of an irrigation district in Almería, Spain. The pipe diameter sizes commercially available for its design are manufactured exclusively in PVC, with an absolute roughness coefficient of 0.0025 mm. The minimum pressure allowable is of 20 m and the pipes’ costs are calculated using a potential function, with a power of 2.06. Its topology is presented in Figure 4.

*Figure 4: Topology of the Balerma network.*

As a result of implementing the Mock Tree methodology on this network, a €2.148 millions discrete design was found. Table 2 presents other reported costs and their respective number of iterations.

*Table 2: Reported costs and number of iterations for the Balerma WDS.*

| **Algorithm** | **Cost (€ millions)** | **Number of iterations** |
|---|---|---|
| Genetic algorithm (Reca & Martínez, 2006) | 2.302 | 10,000,000 |
| Harmony search (Geem, 2006) | 2.601 | 45,400 |
| Harmony search (Geem, 2006) | 2.018 | 10,000,000 |
| Genetic algorithm (Reca et al., 2007) | 3.738 | 45,400 |
| Simulated annealing (Reca et al., 2007) | 3.476 | 45,400 |
| Simulated annealing with taboo search (Reca et al., 2007) | 3.298 | 45,400 |
| Local search with simulated annealing (Reca et al., 2007) | 4.310 | 45,400 |
| Hybrid discrete dynamically dimensioned search *(Tolson, 2009)* | 1,940 | 30,000,000 |
| Harmony search with particle swarm (Geem, 2009) | 2.633 | 45,400 |
| SOGH (Ochoa, 2009) | 2.100 | 1,779 |
| Memetic algorithm (Baños, 2010) | 3,120 | 45,400 |
| Genetic heritage evolution by stochastic transmission | 2,002 | 250,000 |

| | | |
|---|---|---|
| *(Bolognesi, 2010)* | | |
| Differential evolution *(Zheng, 2012)* | 1,998 | 2,400,000 |
| Self-adaptive differential evolution *(Zheng, 2012)* | 1,983 | 1,300,000 |
| OPUS *(Saldarriaga, Páez, Cuero, & León, 2012)** | 2.040 | 957 |
| Mock Tree (this study) | 2.148 | 826 |

*The result reported in the cited paper (€2.106 *millions*) has been recently improved.

**Taichung**

Taichung network was first presented by (Sung, Lin, Lin, & Liu, 2007) and it corresponds to a WDS located in Taichung, Taiwan. The network's topology consists of 20 nodes and 31 pipes organized in 12 loops. For its design there are 13 pipe diameter sizes commercially available, which costs are presented in Table 3. The head-loss equation used is Hazen-Williams with a roughness coefficient (C) of 100 and the minimum pressure for the design scenario is 15 m. Its topology is presented in Figure 5.

*Table 3: Unit costs of Taichung network.*

| Diameter (mm) | Cost (NT Dollar $m^{-1}$) |
|---|---|
| 100 | 860 |
| 150 | 1160 |
| 200 | 1470 |
| 250 | 1700 |
| 300 | 2080 |
| 350 | 2640 |
| 400 | 3240 |
| 450 | 3810 |
| 500 | 4400 |
| 600 | 5580 |
| 700 | 8360 |
| 800 | 10400 |
| 900 | 12800 |

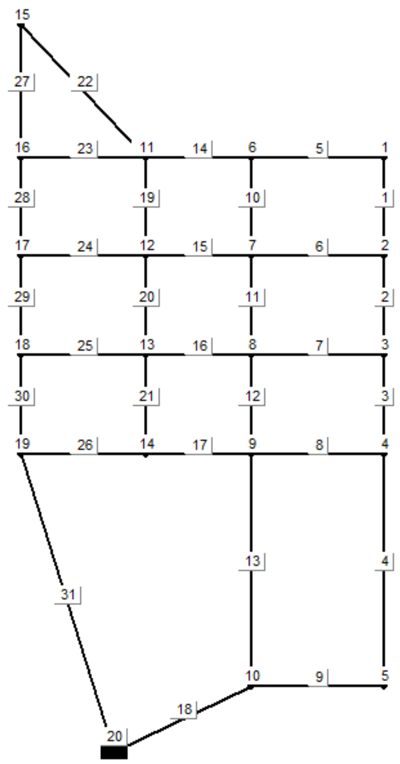


*Figure 5: Topology of Taichung network. The labels show pipe and nodal identification numbers.*

The Mock Tree methodology reached a cost of $8'774,900 after 48 iterations. The pipe diameter sizes in millimetres for this configuration are: 250, 100, 150, 100, 100, 200, 100, 200, 150, 100, 250, 300, 350, 100, 100, 100, 100, 400, 150, 100, 100, 100, 100, 200, 100, 150, 100, 200, 250, 250, and 300 (these diameters are shown in order of pipe identification number).

*Table 4: Reported costs and number of iterations for the Taichung WDS.*

| **Algorithm** | **Cost (NT Dollar)** | **Number of iterations** |
|---|---|---|
| Tabu search *(Sung, Lin, Lin, & Liu, 2007)* | 8,774,900 | Not Available |
| Mock Tree (this study) | 8,966,900 | 48 |

**CONCLUSIONS**

The WDS least-cost design methodology using mock open tree topology herein introduced, considers hydraulic criteria to transform a looped network into an open structure, which is later designed using IPL. This approach differentiates it from the OPUS methodology which first produces a continuous design that is then transformed into a discrete design, through different approximation criteria. Even though these two methodologies have different approximations, they have in common the use of hydraulic principles, unlike metaheuristic algorithms that explore the solution space without considering this kind of criteria.

The methodology significantly reduces the number of iterations and keeps the constructive costs of the network very close to the minimum. In the case of Hanoi the difference results of only 2% with respect to the lowest cost reported in the literature and with a number of iterations four orders of magnitude below.

This methodology clearly proves that considering hydraulic bases together with IPL principles allows the optimization of WDS design to reduce significantly the number of iterations required. The results here found are significantly close to the records, and a little improvement of these would require a really big effort. For this, it is recommended to use this methodology as the basis for new ones but it is not worth it to invest efforts in refining it.

## REFERENCES


Alperovits, E., and Shamir, U. (1977) Design of optimal water distribution systems. Water Resources Research, Vol.13, No.6, pp. 885-900.

Baños, C. G. (2010). A memetic algorithm applied to the design of water distribution networks. *Applied Soft Computing*, 261-266.

Bolognesi, A. e. (2010). Genetic Heritage Evolution by Stochastic Transmission in the optimal design of water distribution networks. *Advances in Engineering Software*, 792-801.

Cunha, M. a. (1999). Water distribution network design optimization: Simulated annealing approach. *J. Water Resour. Plan. Manage.* , 215-221.

Eusuff, M. a. (2003). Optimization of water distribution network design using the shuffled frog leaping algorithm. *J. Water Resour. Plan. Manage.* , 210-225.

Fujiwara, O. and Khang, D. (1990). A two phase decomposition methods for optimal design of looped water distribution networks. *Water Resources Research*, Vol.26, No. 4, pp. 539-549.

Geem, Z. K. (2002). Harmony search optimization: Application to pipe network design. *Int. J. Model. Simulat.* , 125-133.

Geem, Z. K. (2006). Optimal cost design of water distribution networks using harmony search. *Engineering Optimization*, Vol.38, No.3, pp. 259-277.

Geem, Z. K. (2009). Particle-swarm harmony search for water network design. *Engineering Optimization*, Vol.41, No.4, pp. 297-311.

Hernández, D. (2012). Diseño optimizado de submódulos de sistemas de riego localizado a alta frecuencia. Bogotá, DC: Universidad de los Andes.

Kadu, M. R. (2008). Optimal design of water networks using a modified genetic algorithm with reduction in search space. *J. Water Resour. Plan. Manage.* , 147-160.

Lin, M., Liu, G. and Chu, C., (2007) Scatter search heuristic for least-cost design of water distribution networks, *Engineering Optimization*, Vol.39, No.7, pp. 857-876.

Liong, S. and Atiquzzaman, M. (2004) Optimal design of water distribution network using shuffled complex evolution. *Journal of the Institution of Engineers*, Vol.44, No.1, pp. 93-107.

Mohan, S. a. (2010). Optimal water distribution network design with Honey-Bee mating optimization. *J. Water Resour. Plan. Manage*., 117-126.

Mohan, S. a. (2009). Water distribution network design using heuristics-based algorithm. *J. Comput.Civ. Eng.*, 249-257.

Ochoa, S. (2009). Optimal design of water distribution systems based on the optimal hydraulic gradient surface concept. MSc Thesis, dept. of Civil and Environmental Engineering, Universidad de los Andes, Bogotá, Col. (In Spanish).

Perelman, L. and Ostfeld, A. (2007). An adaptive heuristic cross entropy algorithm for optimal design of water distribution systems. *Engineering Optimization*, Vol.39, No.4, pp. 413-428.

Reca, J. and Martínez, J. (2006). Genetic algorithms for the design of looped irrigation water distribution networks. *Water Resources Research*, Vol.44, W05416.

Reca, J., Martínez, J., Gil, C. and Baños, R. (2007). Application of several meta-heuristic techniques to the optimization of real looped water distribution networks. *Water Resources Management*, Vol.22, No.10, pp. 1367-1379.

Saldarriaga, J. (1998 and 2007). *Hidráulica de Tuberías. Abastecimiento de Agua, Redes, Riegos*. Ed. Alfaomega. Ed. Uniandes. ISBN: 978-958-682-680-8.

Saldarriaga, J., Takahashi, S., Hernández, F. and Escovar, M. (2011). Predetermining pressure surfaces in water distribution system design. In *Proceedings of the World Environmental and Water Resources Congress 2011, ASCE*.

Saldarriaga, J., Páez, D., Cuero, P., & León, N. (2012). Optimal power use surface for design of water distribution systems. *XIV Water Distribution Systems Analysis Conference. Tucson, Arizona, USA.*

Savic, D. and Walters, G. (1997). Genetic algorithms for least cost design of water distribution networks. *J. Water Resour. Plan. Manage.*, 67-77.

Sung, Y.-H., Lin, M.-D., Lin, Y.-H. L., & Liu, Y.-L. (2007). Tabu search solution of water distribution network optimization. *Journal of Environmental Engineering and Management*, 177-187.

Suribabu, C. a. (2006). Design of water distribution networks using particle swarm optimization. *Urban Water Journal*, 111-120.

Suribabu, C. (2010). Differential evolution algorithm for optimal design of water distribution networks. *J. of Hydroinf.*, 66-82.

Suribabu, C. (2012). Heuristic Based Pipe Dimensioning Model for Water Distribution Networks. *Journal of Pipeline Systems Engineering and Practice*, 45 p.

Takahashi, S., Saldarriaga, S., Hernández, F., Díaz, D. and Ochoa, S. (2010). An energy methodology for the design of water distribution systems. In *Proceedings of the World Environmental and Water Resources Congress 2010, ASCE*.

Tolson, B. A. (2009). Hybrid discrete dynamically dimensioned search (HD-DDS) algorithm for water distribution system design optimization. *Water Resources Research*, 45 p.

Vairavamoorthy, K. a. (2005). Pipe index vector: A method to improve genetic-algorithm-based pipe optimization. *J. Hydraul. Eng.*, 1117-1125.

Villalba, G. (2004). Optimal combinatory algorithms applied to the design of water distribution systems. MSc Thesis, dept. of Systems and Computation Engineering, Universidad de los Andes.

Wu, I. (1975). Design of drip irrigation main lines. *Journal of Irrigation and Drainage Division*, Vol.101, No.4, pp. 265-278.

Wu, Z. and Simpson, A. (2001). Competent genetic evolutionary optimization of water distribution systems. *Journal of computing in civil engineering*. Vol.15, No.2, 2001, pp. 89-101.

Yates, D., Templeman, A. and Boffey, T. (1984). The computational complexity of the problem of determining least capital cost designs for water supply networks. *Engineering Optimization*, Vol. 7, No.2, pp. 142-155.

Zecchin, A., Simpson, A., Maier, H., Leonard, M., Roberts, A., and Berrisfors, M. (2006) Application of two ant colony optimization algorithms to water distribution system optimization. *Mathematical and Computer Modeling*, Vol.44, No. 5-6, pp. 451-468.

Zheng, F. e. (2012). A Self-Adaptive Differential Evolution Algorithm Applied to Water Distribution System Optimization. *Journal of Computing in Civil Engineering*, 45 p.