# Optimal design of pipes in series with pressure driven demands

Páez, D.[1], Hernandez, D.[2] and Saldarriaga, J.[3]

## ABSTRACT

This paper presents an approach that combines concepts of energy use and the ILP to found near optimal solutions for the optimal design of pipes in series systems in a reduced amount of time. The proposed methodology predefines the head in each node based on known criteria developed by past research on optimal design of looped demand-driven networks. Once the heads are available, the demands are calculated with the demand-pressure function and then the problem is solved as demand-driven with ILP. Taking into account that the resulting design can be unfeasible because of the probable changes in the nodes' heads and therefore in the demand flows, there are needed various iterations of the methodology that explores the head assignation space in an intelligent way. The methodology is tested for different scenarios showing the advantages of this approach.

**Keywords:**

Pipe in series, pressure driven demands, Optimal Hydraulic Gradient Line (OHGL), Integer Linear Programming (ILP)

[1] *Professor, Civil and Environmental Engineering Department, Universidad de los Andes. Researcher, Water Distribution and Sewer Systems Research Center – CIACUA. Email: da.paez27@uniandes.edu.co*

[2] *Researcher, Water Distribution and Sewer Systems Research Center – CIACUA. Civil and Environmental Engineering Department, Universidad de los Andes. Email:: da.hernandez39@uniandes.edu.co;*

[3] *Professor, Civil and Environmental Engineering Department, Universidad de los Andes. Director, Water Distribution and Sewer Systems Research Center – CIACUA. Email: jsaldarr@uniandes.edu.co*

## 1. INTRODUCTION

A pipe in series is a type of water distribution system (WDS) in which there is one reservoir, a set of pipes connected in a lineal way and a set of demand nodes placed on the pipes' junctions. The system is usually called demand-driven whenever the demand on the nodes is independent from the network's hydraulic behavior. Likewise, a pressure-driven model is one in which the demand on each node is a function of the systems pressure.

The optimal designing of WDSs consists in choosing the diameter of each pipe in the system ensuring that the pressure nodes is greater than or equal to a minimum allowable limit, seeking to minimize the system's construction cost. Several methodologies have been used to design demand-driven models. Most of those methodologies consist in heuristics that mimic natural and physical phenomena to explore the solution space e.g Genetic Algorithms (Savic & Waters, 1997; Wu & Simpson, 2001; Reca & Martínez, 2006), Simulated Annealing (Cunha & Sousa, 1999; Reca et al., 2007), Harmony Search (Geem, 2002; Gemm, 2009) and Ant Colony (Zecchin et al., 2006; Ostfled & Tubaltzev, 2008), among others; but some researchers as Ipai Wu in 1975 and Ochoa and Saldarriaga in 2009 have proposed methodologies based on hydraulic/energy concepts as Optimal Pressure Grade Line and Optimal Power Use Surface.

Meanwhile, for pressure-driven models there have been proposed and tested less number of methodologies like Genetic Algorithms (Farmani et al., 2007), Fuzzy Linear Programing (Spiliotis and Tsakiris, 2007) and Recursive Design (González-Cebollada et al., 2011); most of which are applied to design irrigation networks with emitters at their nodes.

This paper presents an approach that combines the mentioned concepts of energy use and Integer Linear Programming (ILP) to found near optimal solutions in a reduced amount of time for pipes in series systems with pressure-driven demands. This research is considered a first step for further methodologies that attempt to solve the WDS optimal design problem for pressure driven demands in more complex networks and based on hydraulics and not in heuristics. It can be especially useful for fire water networks design, WDSs design considering leakage, residential and non-residential plumbing systems design among others.

## 2. PROBLEM FORMULATION

This study deals with pressure-driven demand models with a pipes in series topology. The optimal design can be defined as: Given a layout, lengths of each pipe, topography, connection between pipes and nodes and the minimum pressure requirement, find the diameter combination that implies the minimum construction cost. This combination must obey the mass and energy conservation principles and the minimum pressure requirement on each node (in this study there are not considered other kind of constraints like minimum and maximum velocities). Mathematically, the problem can be expressed as:

$$Minimize\ (C) \tag{1}$$

where $C$ is pipe in series construction cost and is calculated as:

$$C = \sum_{i=1}^{NP} K\, L_i D_i^x \tag{2}$$

where $NP$ is the number of pipes in the series; $L_i$ is the length of pipe $i$; $D_i$ is the diameter of pipe $i$; and $K$ and $x$ are regression parameters for the pipe unitary costs as a function of the diameter. Problem constraints are:

Mass conservation:

$$Q_i = \sum_{\substack{Downstream \\ nodes\ for\ pipe\ i}} \left(BQ_j + EQ_j\right) \qquad \forall i \leq NP \tag{3}$$

where $Q_i$ is the total flow rate at pipe $i$, $BQ_j$ is the base demand at node $j$, and $EQ_j$ is the flow of the emitter at node $j$ and it depends of the pressure on that node as is shown in Equation 4.

$$EQ_j = k_e \cdot h_j^{n_e} \tag{4}$$

where $h_j$ is the pressure head in node $j$, and $k_e$ and $n_e$ are coefficients that describe the emitter characteristics.

Energy conservation:

$$H_j = H_0 - \sum_{\substack{Upstream \\ pipes\ for\ node\ j}} (hf_i + hm_i) \qquad \forall j \leq NN \tag{5}$$

where $H_j$ is the total head in node $j$, $H_0$ is the total head at the reservoir, $hf_i$ is the friction loss in pipe $i$; $hm_i$ is the minor loss in pipe $i$ and $NN$ is the number of nodes. For this study friction losses are calculated with Darcy-Weisbach equation.

Minimum pressure in demand nodes:

$$H_j \geq H_j^{(\min)} \qquad \forall j \leq NN \tag{6}$$

where $H_j^{(\min)}$ is the minimum head required in node $j$ which corresponds with the minimum allowable pressure.

Pipe diameters can only take discrete values belonging to commercial diameters set $\Phi_D$:

$$D_i \in \Phi_D \qquad \forall i \leq NP \tag{7}$$

It should be noticed that the flow in each node is not known before the design as they depend of the pressure on each downstream node, and for that reason IPL cannot be used directly to find the global optimum of the problem.

## 3. OPTIMUM HYDRAULIC GRADE LINE FOR A PIPE IN SERIES

As well as I-pai Wu (1975) and later Ochoa and Saldarriaga (2009) established, the minimum cost design usually develops a parabolic hydraulic gradient line (HGL). In order to establish the behavior of the quadratic equation of the hydraulic gradient, there must be known three points that describes the parabolic function. In the case of the hydraulic gradient, the three points are:

Hmax: is the available head for the entire network and as it is the head at the reservoir, it is placed at abscissa $d = 0$.

Hmin: is the minimum head for the critical node which is either the final node or a node that will have a total head closer to the minimum because of its elevation. As it defines the final node, it is placed at abscissa $d = d_{total}$.

Hsag: corresponds to the head in the point of maximum curvature in the hydraulic gradient line. This point is defined by the Sag which is a percentage of the difference between Hmax and Hmin line and it will determine the Hsag as shown in Figure 1. It is always placed at abscissa $d = d_{total}/2$.

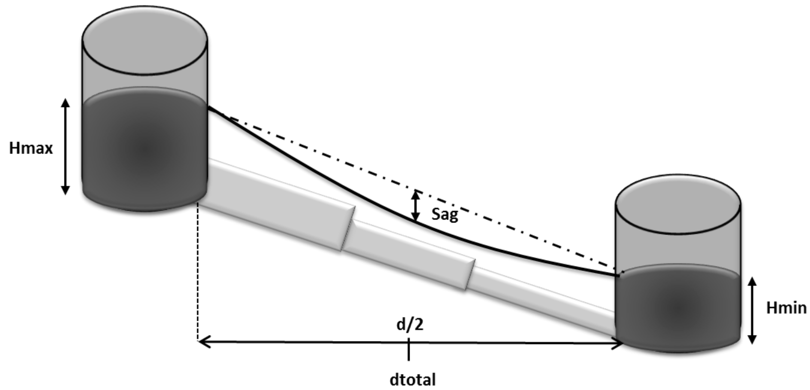


**Figure 1. HGL goal, based on three known points.**

As shown in Figure 1 it can be seen that there is a straight line corresponding to the case when the hydraulic gradient line is linear. When the Sag is 0 the gradient will be equal to the straight line, but when the sag is different to 0 the head in the middle of the pipe system will be equal to the head in the middle point for the straight line minus the Sag multiplied by the available head in the system.

This means that the objective hydraulic gradient line could be found by this expression:

$$HGL_j = \alpha d^2 + \beta d + \gamma \quad [8]$$

where:

$$\alpha = 4S \frac{(H_{max} - H_{min})}{{d_{total}}^2}$$

$$\beta = -(1 + 4S) \frac{(H_{max} - H_{min})}{d_{total}}$$

$$\gamma = H_{max}$$

$HGL_j$ is the objective head on node $j$ placed at a distance $d$ from the reservoir; S is the selected sag and $H_{max}$ and $H_{min}$ are the heads at the mentioned points.

## 4. METODOLOGY

The proposed methodology is show in Diagram 1 and explained above:

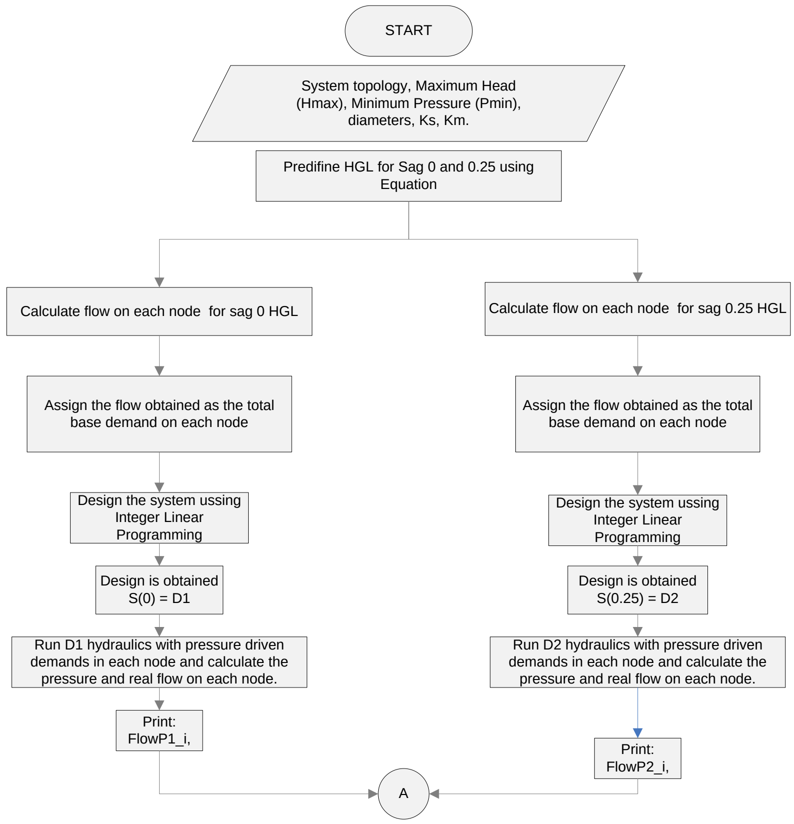


**Diagram 1. Methodology.**

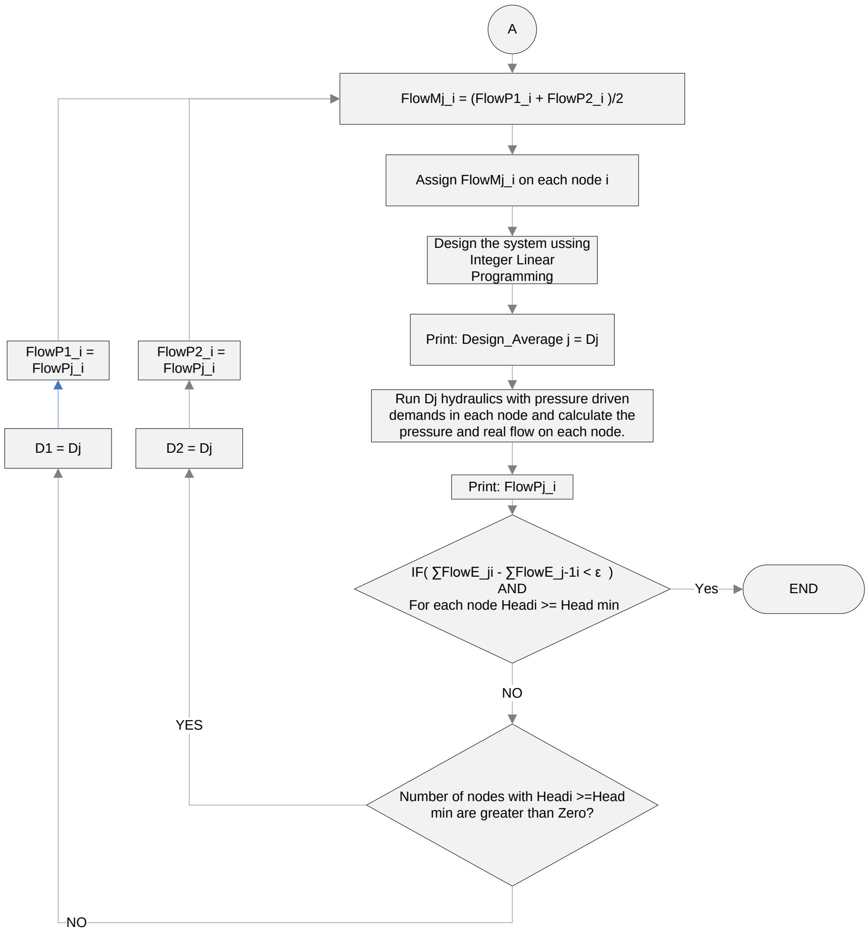


**Diagram 2. Methodology.**

### 4.1 Predefine Hydraulic Gradient Line

In order to calculate the HGL is important determine the sag that will be used. It can be proved that the sag have a validity range between 0 and 0.25. When design is based on the HGL with a 0 sag the system meets the minimum pressure restriction but generates high constructive costs as it overestimates the emitter flows. Whereas when is based on a 0.25 sag the design has low constructive cost but does not meet the minimum pressure because of its subestimation of emitter flows.

Considering that behavior, the methodology looks for an average between those two designs looking to accurately estimate the emitter flows and therefore the flow rate in each pipe.

### 4.2 Designs with Integer Linear Programming

Once the flow rate in each pipe is supposed by using the results of the last step, the optimum design of that system can be achived with the following ILP formulation:

Define $N$ as the set of nodes in the network, $\Phi_D$ as the set of available diameters and $X_{ijd}$ as binary decision variables described by Equation :

$$X_{ijd} = \begin{cases} 1 & if\ for\ the\ pipe\ from\ node\ i \in N\ to\ j \in N\ was \\ & assigned\ the\ diameter\ d \in \Phi_D \\ 0 & otherwise \end{cases} \qquad [9]$$

Also define $H_i$as auxiliary decision variables that represent the total head in the node $i \in$ N. Then the objective function is:

$$\sum_{i \in N} \sum_{j \in N} \sum_{d \in D} C_{ijd} \cdot X_{ijd} \qquad [10]$$

were $C_{ijd}$ is the cost of assigning a diameter $d \in D$ in the pipe that goes from node $i \in N$ to the node $j \in N$. Finally the constraints for the ILP problem are:

<u>Constraints:</u>

- Constraint of minimum allowable pressure and its consequent total head defined by Equation 6.
- Constraint that ensures the conservation of energy for each pipe. The total head at node $j \in N$ downstream the node $i \in N$ will be equal to the total head in node $i$ minus the total head losses produced in the pipe from $i \in N$ to $j \in N$ when a diameter $d \in D$ is assigned to that pipe:

$$H_j = H_i - \sum_{d \in D} dp_{ijd} \cdot X_{ijd} \qquad \forall\, i \in N, \forall\, j \in N \mid w(i,j) = 1 \qquad [11]$$

  were $H_j$ is the head in the downstream node, $H_i$ the head in the upstream node, $dp_{ijd}$ is the parameter of total head losses that occurs in pipe from node $i \in N$ to $j \in N$ when a diameter $d \in D$ is assigned, and $w(i,j)$ is a function that returns a 1.0 when the pipe that goes from $i$ to $j$ acctually exists ad a 0.0 otherwise.
- Constraint that ensures that only one diameter is assigned to each pipe:

$$\sum_{d \in D} X_{ijd} = 1 \qquad \forall\, i \in N, \forall\, j \in N \mid w(i,j) = 1 \qquad [12]$$

This formulation was implemented in the program Xpress IVE. The Xpress-Optimizer features sophisticated, robust multi-threaded algorithms to quickly and accurately solve linear problems (LP).

After this step the designer will have two different designs, the design obtained from Sag 0 ($D_1$) and the design obtain from Sag 0.25 ($D_2$). It is important to mention that, the design $D_1$ will be more expensive than the design $D_2$; but on the other hand, the design $D_1$ will be feasible hydraulically and $D_2$ probably won't. This is verified in the next step.

### 4.3 Hydraulic execution with pressure driven demands

As explained before, the designs $D_1$ and $D_2$ were obtained from constant demands, so it is necessary to verify their hydraulic behavior when they are modeled with pressure driven demands. In case that the design $D_2$ results in a feasible design the algorithm ends and the final design will be $D_2$, otherwise the process continue to next step.

The flows for each node calculated with the hydraulic execution of $D_1$ and $D_2$ considering pressure-driven demands are stored. For $D_1$ the flow is $Flow1_i$ and for $D_2$ is $Flow2_i$ , $i$ is the node ID.

### 4.4 Iteration process

1. Using the flows for each design ($Flow1_i$ and $Flow2_i$) after hydraulic execution, a new estimation of the flow for each node is calculated with Equation 13 for each node $i$.

$$Flow_{mi} = \frac{FlowE_{1i} + FlowE_{2i}}{2} \qquad [13]$$

where $Flow_{mi}$ corresponds to the new flow in the node $i$ for the next design called Dm (the middle design).

2. Assign $Flow_{mi}$ as a constant demand on each node.

3. Design the new middle system with LP using steps described at section 4.4.

4. Verify the hydraulic performance for design $D_m$ obtaining the actual heads and total flow in each node.

5. Restore $D_1$ or $D_2$: The designer gets to this step due to the unfeasibility of $D_m$ and/or because a cheaper design is expected by reducing even more the supposed flow for each node. In this step the designer has to observe the number of nodes under the minimum pressure in $D_m$:

$$If\ number\ of\ nodes\ under\ minimum\ pressure\ > 0\ \rightarrow D_2 = D_m$$

$$Else, if\ number\ of\ nodes\ under\ minimum\ pressure = 0\ \rightarrow D_1 = D_m$$

In the presented conditional of this step, it can be observed that it starts a bisection process. After replacing $D_m$ in $D_2$ or $D_1$, the designer has to go back to step 1 and repeat the process until the differences between the flows supposed by the last $D_m$ and the actual $D_m$ are negligible.

## 5. RESULTS

The proposed methodology was tested on 4 systems with a similar layout (15 pipes in series) but with differences in topography and base demand on the nodes. Network 1, has no base demand and flat topography, Network 2 has a 240 Lps based demand

and flat topography, Network 3 has no base demand and a topography presented on Figure 2, and Network 4 has the same topography and a 240 Lps base demand. The total head at the reservoir is 35.0 m and the minimum allowable pressure for the nodes is 10.0 m. The available diameters are 50, 75, 100, 150, 200, 250, 300, 350, 400, 450, 500, 600, 750, 800 and 1000 mm for networks 1 and 3 but considering the base demand of networks 2 and 4, diameters of 1200, 1400 and 1500 were added to the list. The roughness of the pipes is $1.5 \cdot 10^{-6}$ m and cost parameters were $K = 0.015$ and $x = 1.46$. There were no minor losses considered on these networks.

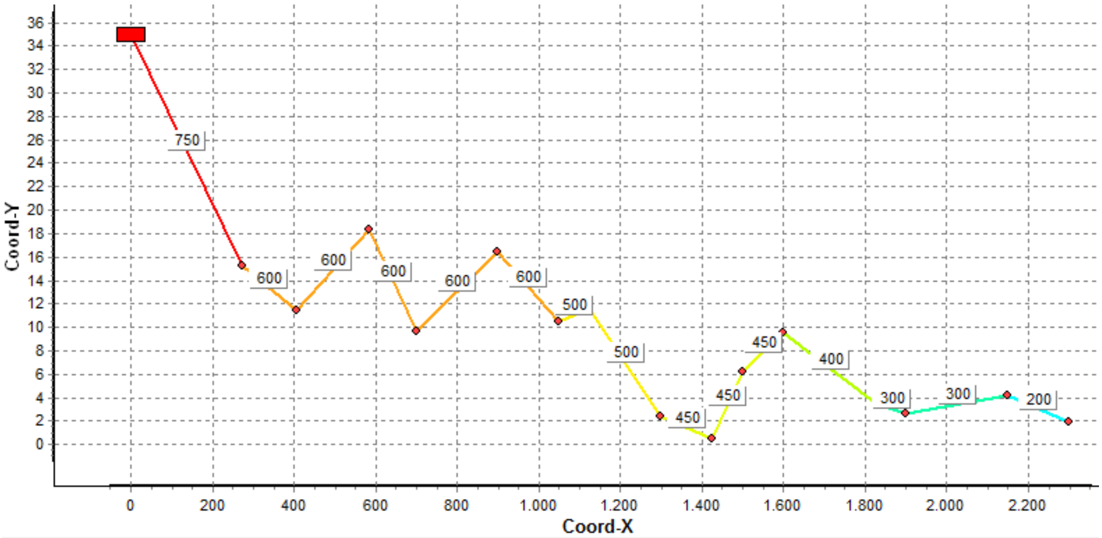


**Figure 2. Topography configuration for Network 3 and Network 4 and pipes' lengths for the four study cases.**

The comparison was made with the SOGH methodology proposed by Ochoa (2009) which is the methodology that presents the criterion of parabolic HGL to minimize constructive costs. As the Sag mentioned above is a free parameter of the parabolic equation, a design was made for different sag values.

On the other hand, the proposed methodology was implemented in REDES software for hydraulic executions developed by CIACUA as well as Xpress-IVE for ILP problems' solution. The results are presented on the following figures:

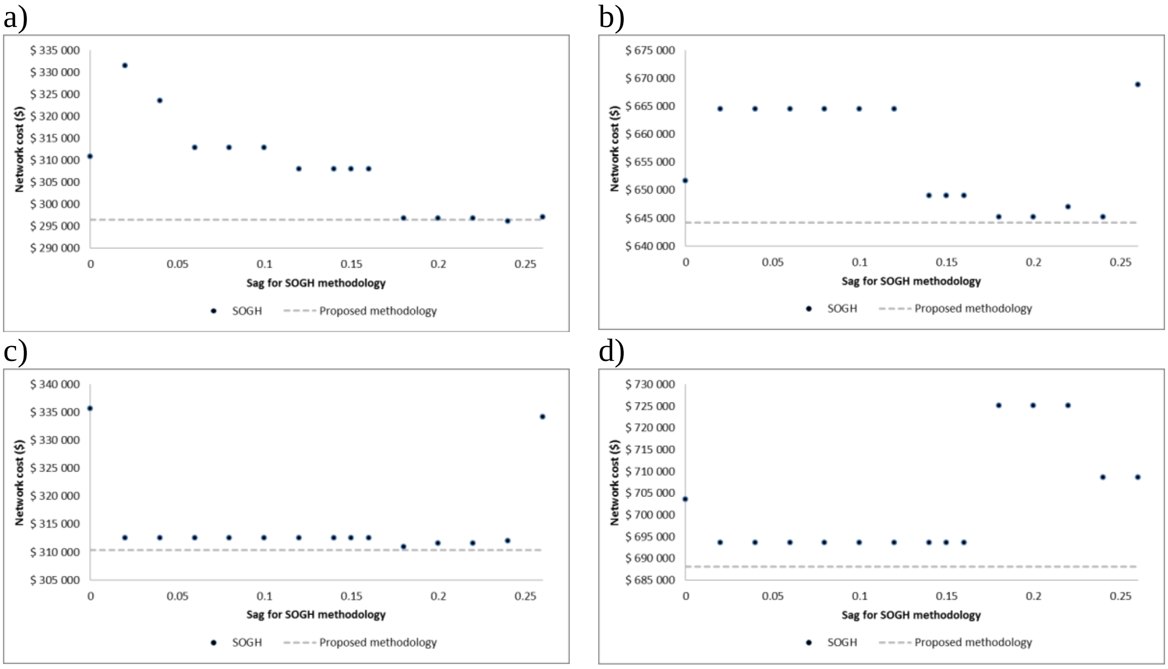


**Figure 3. Costs results for the four study cases. a) Network 1, b) Network 2, c) Network 3 and d) Network 4.**

It can be seen that the proposed methodology achieves designs with less constructive costs than SOGH for three of the four study cases, and for the Network 1, a design that costs 0.13% more than the better result found with SOGH. That means that the ILP methodology is actually finding near optimal solutions in all the networks.

The next criterion that is compared is the computational time required to find that designs. For the Network 1 the best design found by SOGH (sag = 0.24) required 43 hydraulic executions of the system; for Network 2 the best design can be achieved with the sag values 0.18, 0.2 and 0.24 requiring 40 hydraulic executions; for Network 3 the minimum cost sag was 0.18 with 88 hydraulic executions; and finally the Network 4 best design with SOGH required 71 hydraulic executions with a 0.02 sag but that same design can be found using sags between 0.02 and 0.16, each value requiring different number of hydraulic executions.

It should be noticed that the previous number of executions required by SOGH methodology are actually the executions required if you know a priori the optimal sag, but it is a difficult task as it depends on the demand distribution among the system, and as it is pressure-driven it is not known before the design. Therefore the SOGH methodology required actually 568 hydraulic executions for Network 1, 591 for Network 2, 1019 for Network 3 and 1345 for Network 4, which were used in 14 different designs for each network with sags varying from 0.02 to 0.26.

On the other hand the computational time spent by the ILP methodology is composed by the time assigning the HGL, which is negligible, the time defining each ILP formulation, which requires the calculation of each $dp_{ijd}$ (total head losses that occurs in pipe from node $i \in N$ to $j \in N$ when a diameter $d \in D$ is assigned), the ILP problems' solving and the hydraulic executions required after each ILP formulation.

The $dp_{ijd}$ computing can be done by assigning to the entire system the diameter $d$ and then executing the hydraulics reading the head losses on the pipes, and repeating that process for each available diameter. The ILP solving lasted less than 0.1 seconds on a Intel Core i5 processor with 3.0 GB RAM Memory using Xpress-IVE software, so it is also negligible when compared with the hydraulic executions.

Therefore the proposed methodology required 80 hydraulic executions for the Network 1, but 75 of those were executions with constant demand on the nodes as there were used just for the computing of the $dp_{ijd}$ and only 5 executions were actually done with the system with pressure-driven demands. Considering the way in which the pressure-driven demand models are executed with the Gradient Method programmed in EPANET (Rossman, 2000) and also in REDES software, the 75 hydraulic executions with constant demand plus the 5 executions with pressure-driven demands are barely more time demanding than the 43 executions with pressure driven demand spent by SOGH.

In the case of Network 2, the ILP methodology required 36 hydraulic executions with constant demands and 2 executions with pressure-driven demands, resulting in fewer executions than the best design accomplished by SOGH. For Network 3, 60 hydraulic executions with constant demands and 4 executions with pressure-driven demands were required. Finally for Network 4 were spent 54 hydraulic executions with constant demands and 3 executions with pressure-driven demands.

It means that the proposed methodology can achieve near optimal designs in a reduced amount of time considering pressure-driven demands using hydraulic criteria for the definition of the HGL and ILP for the consequent diameters selection.

## 6. CONCLUSIONS

A design methodology that uses hydraulic criteria to predefine an objective hydraulic grade line and Integer Linear Programming to design a pressure-driven system, was presented and tested on four study cases with a pipes' in series topology, showing its benefits in terms of the quality of the solutions (reduced constructive cost) and the amount of computational time required (reduced number of hydraulic executions).

This study is a first step to develop further methodologies that solves the WDS optimal design problem for pressure driven demands in more complex networks. It can be especially useful for fire water networks design, WDSs design considering leakage, residential and non-residential plumbing systems design among others.

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